Angular correlation properties of multiply scattered light in random media with buried objects

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ABSTRACT

Effect of objects buried in highly dense media on the properties of angular intensity correlation of the scattered light is investigated by means of a theoretical approach and numerical simulations. In the case of an object located just behind the scattering medium, a Fourier transform relation between the angular correlation function and the intensity distribution in the illumination plane where the object exists is derived theoretically for transmitted light. When the object is buried in the medium, such a boundary condition is not satisfied, and it was found from the simulation results that the effect of the object on the angular correlations can be observed more strongly for the object being somewhat deep rather than near the output surface. For reflected light, on the other hand, the influence of the object on the angular correlation function becomes small because of a scattering component not reaching the object.

Keywords: intensity fluctuations, angular correlation, multiple scattering, object detection, diffusion approximation, short- and long-range correlations

1. INTRODUCTION

The problem of detecting objects hidden in highly dense media is quite complex because of strong multiple scattering of light. Recently, a number of studies on this subject have been performed by means of some effective methods, such as a pulse propagation, a photon density wave and a low coherence technique, and are expected to be applied in biomedical imaging and other optical measurements.[1] As one of objects for such measurements, let us consider an object hidden in static scattering systems like a powder bed and a white paint. For such systems, the randomness is never averaged spontaneously, and, thus, the intensity pattern fluctuates randomly in the detecting field like laser speckles. Therefore, it is desired to obtain some information of the object from the statistical properties of the intensity fluctuations.

As a relation between an incident field and intensity fluctuations in the detecting plane, an optical memory effect is well known in this field,[2] in which a change of the incident angle induces the translation of the fluctuated intensity pattern in the far-field plane. This fact indicates that the characteristic of the incident field is reflected in the angular correlation function, even after the scattered light propagates diffusively in a highly dense medium. In the present paper, the angular correlation function for an arbitrary illumination field is derived theoretically from the diffusion approximation in transmitted and reflected light by considering only a short-range contribution, which is derived by assuming the independence of all scattering paths and brings about the optical memory effect. The scattering geometries are shown in Fig. 1.

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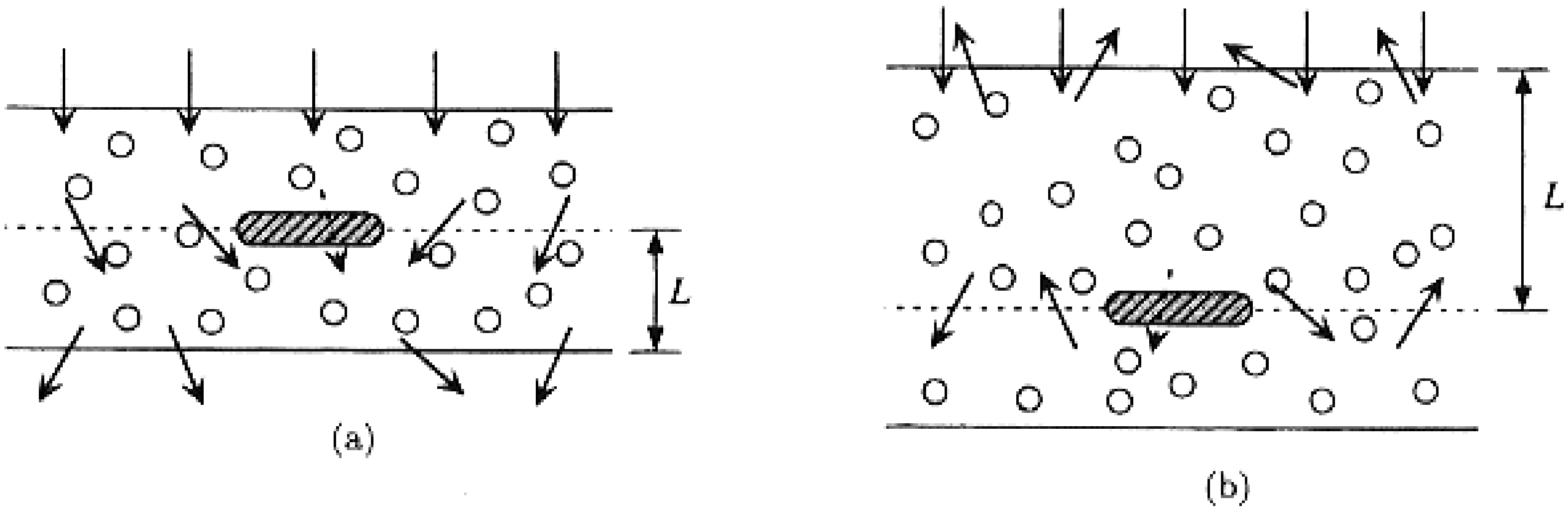

(a)

(b)

Figure 1. Scattering geometries for (a) transmitted and (b) reflected light.

2. ANGULAR INTENSITY CORRELATION: THEORY

Under the approximation that all scattering paths in the medium are mutually independent, the angular correlation function is given by

$$\begin{aligned} C(\Delta q) &\equiv [\langle I(\boldsymbol{q})I(\boldsymbol{q}+\Delta\boldsymbol{q})\rangle - \langle I\rangle^2]/\langle I\rangle^2 \\ &= \frac{1}{\langle I\rangle^2}\left|\frac{1}{A^2}\iint \langle I_i(\boldsymbol{\rho})\rangle\langle|G(\boldsymbol{\rho},z;\boldsymbol{\rho}',z')|^2\rangle \exp(-i\Delta\boldsymbol{q}\cdot\boldsymbol{\rho}')d\boldsymbol{\rho}d\boldsymbol{\rho}'\right|^2, \end{aligned} \tag{1}$$

where $\Delta\boldsymbol{q}$ is a transversal component of the wave vector with $\Delta q \approx k\Delta\theta$ for a small angular shift, θ being a difference between the scattering angles, and A, $\boldsymbol{\rho}$ and $\boldsymbol{\rho}'$ denote the illumination area, and vector coordinates in the output and the illumination planes, respectively. $\langle I_i\rangle$ is an average intensity distribution in the illumination plane and is set to be zero in an area occupied by the object. G represents the Green function satisfying a wave equation and its mean square modulus $\langle|G|^2\rangle$ satisfies a diffusion equation under this approximation.

For transmission geometry, $\langle|G|^2\rangle = 0$ at the boundaries $z' = 0$ and L, where L is the sample thickness, and Eq. (1) is reduced as

$$C(\Delta q) \approx \left[\frac{\Delta qL}{\sinh(\Delta qL)}\right]^2 \left|\int \langle \overline{I}_i(\boldsymbol{\rho}')\rangle \exp(-i\Delta\boldsymbol{q}\cdot\boldsymbol{\rho}')d\boldsymbol{\rho}'\right|^2, \tag{2}$$

where $\langle\overline{I}_i\rangle$ is a normalized average intensity distribution in the illumination field. For reflection geometry, although an additional term appears in Eq. (2) because of the scattering component which does not reach the illumination plane where the object is located, the Fourier transform relation between the angular correlation function and the intensity distribution in the illumination plane is also found.[3] Therefore, the information of the object is reflected in the angular correlations and a behavior of the angular correlation function changes depending on both sizes of the illumination area and the object.

3. NUMERICAL SIMULATION AND RESULTS

Numerical simulations used to obtain the angular correlation functions of the scattered intensity was proposed by Edrei, *et al.*[4] and extended by Sangu, *et al.*[5] In this simulation, the scattering and free propagation between the scattering particles are expressed by S-matrix assigned to each site and complex amplitudes are calculated until a steady state is reached. For simplicity, the simulation is performed only in a two-dimensional case. However, the angular correlation properties of scattered light do not depend on the sample dimensionality. Some parameters are fixed in this simulation; the sample thickness $L = 10l$ and width $W = 128l$, l being the scattering mean free path, Ioffe-Regel product $kl = 500$, and all scatterings are isotropic. A completely absorbing object is assumed as hidden objects and realized by substituting zero amplitude in the area occupied by the object or by setting all elements of the S-matrices occupied by the object inside the medium at zero. The angular correlation in such a case is derived from Eq. (2) as

$$C(\Delta q) \approx \left[\frac{\Delta qL}{\sinh(\Delta qL)}\right]^2 \left[\frac{\sin(\Delta qW/2) - \sin(\Delta qw/2)}{(\Delta q/2)(W-w)}\right]^2, \tag{3}$$

where w is the size of the object. In Fig. 2, the angular correlation functions are plotted for the plane wave incidence with and without the object together with a theoretical curve given by Eq. (3), where the object size is set at $w = 16l$. The correlation peaks around $\Delta\theta = 0$ are due to the speckle spot, while the presence of the object affects another correlation peak around $\Delta\theta \sim 8$ mdeg, which is not observed for the case without the object. The correlation functions calculated by numerical simulations have somewhat larger values than that by the theoretical estimation. This is the long-range contribution which is ignored in our theoretical estimation and becomes large with an increase in the sample thickness.

When the object is buried in the highly dense medium as illustrated in Fig. 1, the boundary condition which was applied in Eq. (2) does not hold correctly and, thus, a part of the scattered light passing through the illumination plane reaches again the absorbing object. This induces an anomalous property in the angular correlation function. In Fig. 3, the angular correlation functions are plotted for three depths of the buried object, $d = 2l$, $5l$, and $9l$. For $d = 5l$, in spite of the middle depth of the buried object, the correlation peak relating to the presence of the object

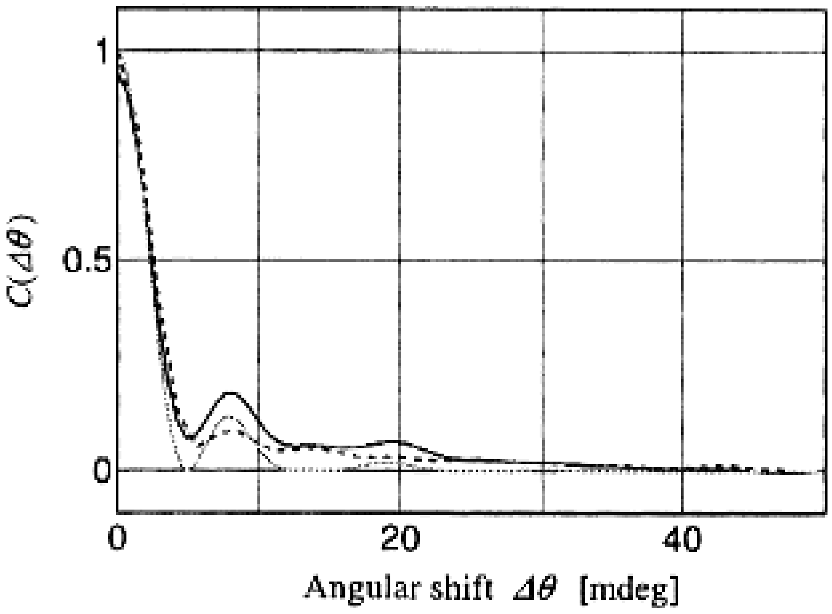

Figure 2. Angular correlation functions for the transmitted light. The solid, dotted and gray curves represent the cases with and without the object, and the theoretical one, respectively.

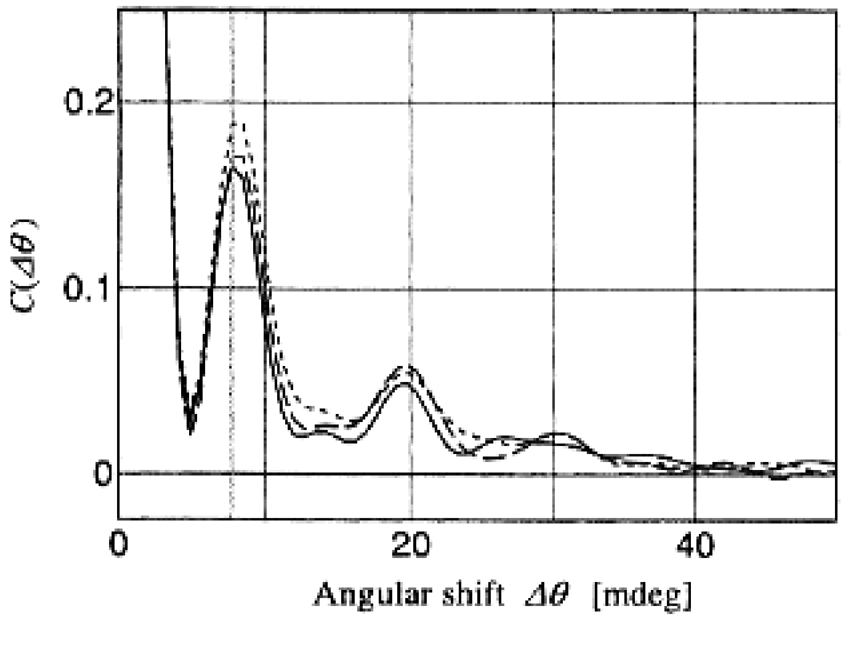

Figure 3. Angular correlation functions for the transmitted light. The solid, dotted and dashed curves stand for the object depths of $d = 2l$, $5l$ and $9l$, respectively. The vertical gray line denotes $\Delta\theta \sim 7.73$ mdeg.

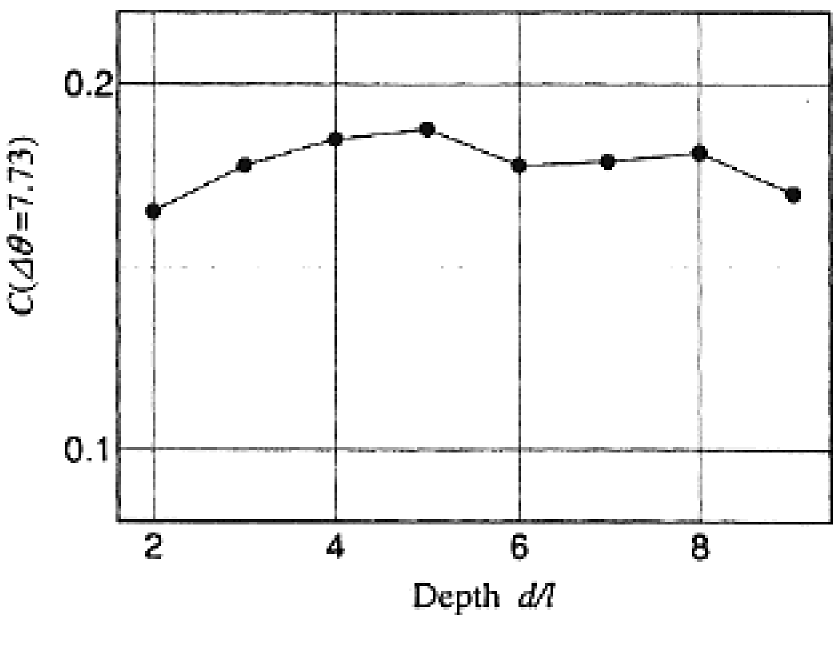

Figure 4. Correlation peak height for the transmitted light at the angular shift $\Delta\theta \sim 7.73$ mdeg against the object depth.

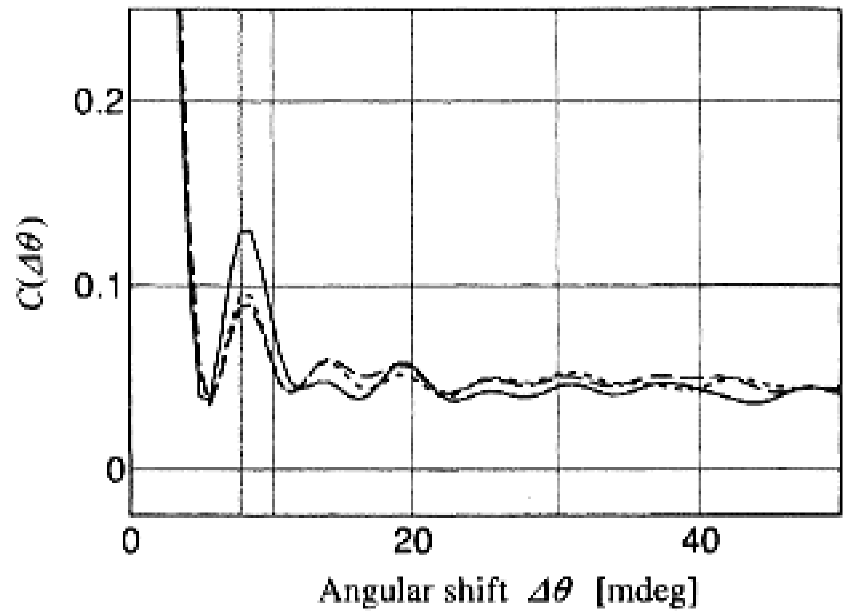

Figure 5. Angular correlation functions for the reflected light. The solid, dotted and dashed curves stand for the object depths $d = 2l$, $5l$ and $9l$, respectively. The vertical gray line denotes $\Delta\theta \sim 7.73$ mdeg.

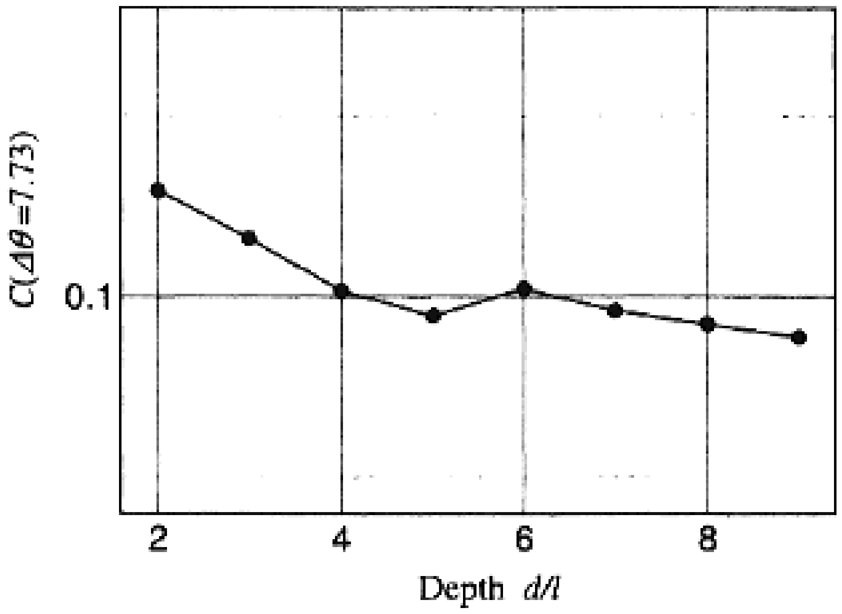

Figure 6. Correlation peak height for the reflected light at the angular shift $\Delta\theta \sim 7.73$ mdeg against the object depth.

shows the highest value in the three curves. The peak height at $\Delta\theta \sim 7.73$ mdeg is plotted against the depth of the object in Fig. 4. Although the variation of the correlation is relatively small, the correlation values increase with the depth of the object in the range of small depth between $d = 2l$ and $5l$, and then, decrease with a further increase in the depth. From this result, it is found that the influence of the absorbing object appears more strongly for the object in a moderate depth rather than near the detecting surface.

Next, we turn the viewpoint to the reflection geometry. Figure 5 shows the angular correlation functions of the reflected light for the same depths with Fig. 3. The reflected light includes the scattering component that has not passed through the illumination plane and, thus, the change in the correlation function due to the object becomes less detectable than for the transmitted light. In Fig. 6, the correlation peak height at $\Delta\theta \sim 7.73$ mdeg is shown for the reflected light. In this case, the peak values decrease monotonously with an increase in the object depth.

4. CONCLUSIONS

As one of methods for detecting buried objects in highly dense media, the properties of intensity correlations of speckle-like intensity fluctuations have been investigated by a theoretical approach and numerical simulations. From the theoretical study, it is shown that the correlation peak that depends on the presence of the completely absorbing or opaque object behind the scattering medium appears in both of the transmitted and reflected light. In the case of the object located inside a medium, one may consider that an object distant from the output surface is less detectable. Our simulation results, however, show that the object placed near the output plane does not always affect the property of the angular correlation more strongly. Because of the boundary effect, the influence of the object on the angular correlation function becomes strongest for the middle depth of the object position in the transmission geometry. For the reflection, this anomaly was not observed, but the scattering component not reaching the illumination plane makes weak the effect of the object. Further theoretical studies for this boundary effect are now in progress.

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