Numerical analysis of enhanced backscattering from random fractal media

Jun Uozumi and Takamitsu Saito

Research Institute for Electronic Science, Hokkaido University
Sapporo, Hokkaido 060-0812, Japan

ABSTRACT

Enhanced backscattering from random fractal media is investigated by means of Monte Carlo simulations based on a randomized Menger sponge model of the media. Dependence of the coherent peak on the fractal dimension D is obtained for $1.5 \leq D \leq 2.79$, and it is found that the slope of the peak shoulder in a log-log plot increases with D until $D = 2.5$ and then decreases for a further increase in D. This behavior is elucidated on the basis of probabilities of the scattering free path, the number of scattering in a multiple scattering path, and the separation of two end particles in a scattering path.

Keywords: Enhanced backscattering, random fractal media, Monte Carlo simulation, fractal dimension

1. INTRODUCTION

When strong multiple scattering media such as a dense collection of particles are illuminated by the coherent light, appreciable enhancement of scattered intensity is observed in the retroreflection direction. Since the first experimental observation by Kuga and Ishimaru,[1] extensive studies have been devoted to this phenomenon of enhanced backscattering.[2] Recently, influences of fractality of scattering media on the coherent peak have attracted attention.[3,4] For inhomogeneous random media such as fractal aggregates, however, theoretical analyses based on a diffusion approximation for photon propagation are not applicable and, therefore, a computer simulation approach was used by Ishii et al.[5] In this simulation, the fractality of the scattering media is represented by the probability density function (PDF) of scattering free paths which was derived analytically, and a sequence of random numbers obeying this density determines a multiple scattering path in the fractal media. Owing to a restriction of this density function, however, this approach is effective in the limited range of $2 < D < 3$ and, unfortunately, the fractal dimension of the aggregates used in the past experiments[3,4] is out of this range.

In the present paper, another simulation study employing a different approach is performed to overcome this problem. In the new approach, fractal objects with desired fractal dimensions are numerically generated on a computer memory, and the photon migration is traced by a Monte Carlo method in the object. This approach has complementary features to the former one in various aspects, and can cover the range of $0 < D < 3$.

2. SIMULATION METHOD

2.1. Fractal media

In the new approach, details of fractal media must be specified using a certain fractal-generating algorithm prior to numerical scattering experiments. We examined several different algorithms for generating fractals, and decided to employ the structures produced by generalizing and randomizing Menger sponges.[6] This fractal has advantages that the fractal dimension is almost constant from its lower limit to the upper of a given scaling range, and that arbitrary fractal dimensions in the range $0 < D < 3$ can be specified. As a trade-off for these advantages, however, we loose the connectivity between particles in the fractal, which is an inherent property of fractals such as aggregated particles. The point is that the modified Menger sponge provides us with a well-behaved model of mass fractals with desired fractal dimensions.

The modified Menger sponge is generated by the following procedure. Assume a cube of size M, and divide it into a^3 small cubes by dividing each side into a equal segments. Take b out of a^3 small cubes at random and regard each cube as including at least one particle. This step of division and selection is applied again to each of b small

Send correspondence to J.U.: E-mail: uozumi@es.hokudai.ac.jp

Part of the OSJ/SPIE Conference on Optical Engineering for Sensing and Nanotechnology (ICOSN '99) • Yokohama, Japan • June 1999
SPIE Vol. 3740 • 0277-786X/99/$10.00

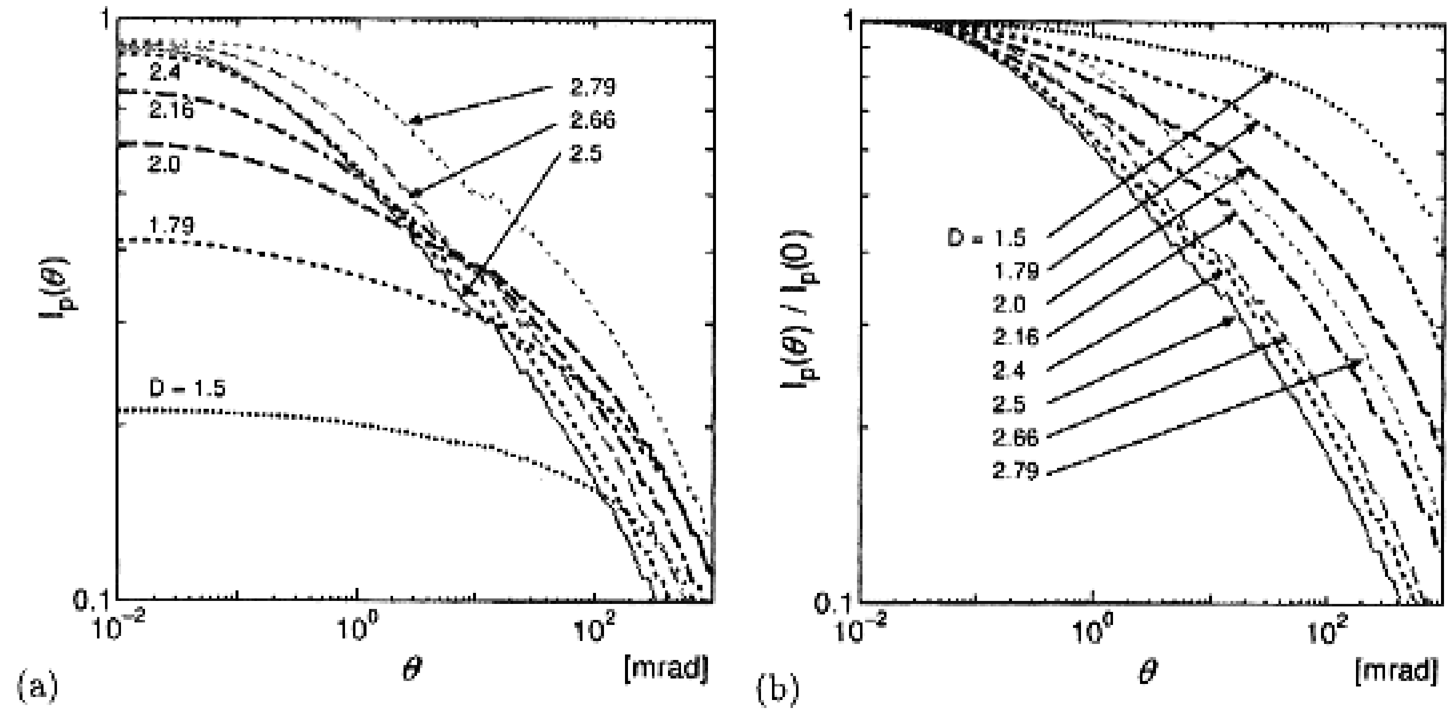

Figure 1. (a) Intensity distributions of the coherent peaks and (b) their normalized forms by the maximum values as a function of the backscattering angle θ for several fractal dimensions in the range of $1.5 \leq D \leq 2.79$.

cubes, and this procedure is repeated until the divided cubes are reduced to a unit cube corresponding to an element particle of the fractal. The fractal dimension of this structure is given by $D = \log b / \log a$. In the simulation, the fractal dimension is varied by fixing the parameters $a = 4$ and $M = 256$ while changing the parameter b. As a result, particles are located on a cubic lattice, and the diameter of particles is equal to the lattice spacing.

The restriction of $M = 256$ comes from the memory size of a computer used in the simulation, and is not sufficient for simulating scattering processes. However, the object can be enlarged effectively by employing an algorithm based on the self-similarity of the fractal. That is, each particle of the generated Menger sponge is identified with the entire sponge and, thus, the effective object size is extended to $M_e = M^2 = 65536$.

2.2. Ray tracing and the calculation of backscattering intensity

To reduce the simulation time, we simplify the problem by assuming that the light is scalar waves and is scattered isotropically by each particle. The particle size is set to 0.1 μm, while $\lambda = 0.386\,\mu$m, the value of $\lambda = 0.5145\,\mu$m in water, is used for the wavelength of light. The scattering process is simulated in the following way. A ray of light is incident normally on one of six surfaces of the cube defining the medium and proceeds until impinging one of particles. Then, a scattering angle is determined from random numbers so that the probability of direction is uniform over the entire solid angle, and the ray tracing is continued. This procedure is continued until the ray exits from the medium or the number of scattering n_s exceeds 400. In self-similar algorithm for extending object size, the ray is traced also in the *inside space* of particle until it exits to the *outside space* or until n_s reaching the limit. Only when the ray exits from the incident surface, the coordinates of the first and last scattering particles are recorded, from which backscattering intensity distributions are calculated by

$$I(\theta) = \frac{N_s + N_m + \sum_{n=1}^{N_m} \cos[(\boldsymbol{k}_i + \boldsymbol{k}_o)(\boldsymbol{r}_{fn} - \boldsymbol{r}_{ln})]}{N_s + N_m} = 1 + I_p(\theta), \tag{1}$$

where θ is a backscattering angle with the origin at the retroreflection direction, N_s and N_m are the numbers of rays exiting after single and multiple scatterings, respectively, $\boldsymbol{k}_i$ and $\boldsymbol{k}_o$ are wave vectors of the incident and exiting rays, respectively, $\boldsymbol{r}_f$ and $\boldsymbol{r}_l$ are the coordinates of the first and last scattering particles, and the subscript n is a running index for exiting rays. The unity in the rightmost side of eq. (1) represents a constant background intensity, while the second term $I_p(\theta)$ indicates the coherent intensity peak.

3. RESULTS AND DISCUSSION

3.1. Intensity distribution of the coherent peak

The backscattering peak intensity $I_p(\theta)$ was calculated and is shown in Fig. 1 for the range of $1.5 \leq D \leq 2.79$ of the fractal dimension D. From Fig. 1(a), it is seen that the maximum intensity increases monotonously with the fractal dimension. Since $\boldsymbol{k}_o = -\boldsymbol{k}_i$ at the origin ($\theta = 0$), it follows from eq. (1) that $I_p(0) = \frac{N_m}{N_s + N_m} = \frac{R}{1+R} = 1 - \frac{1}{1+R}$,

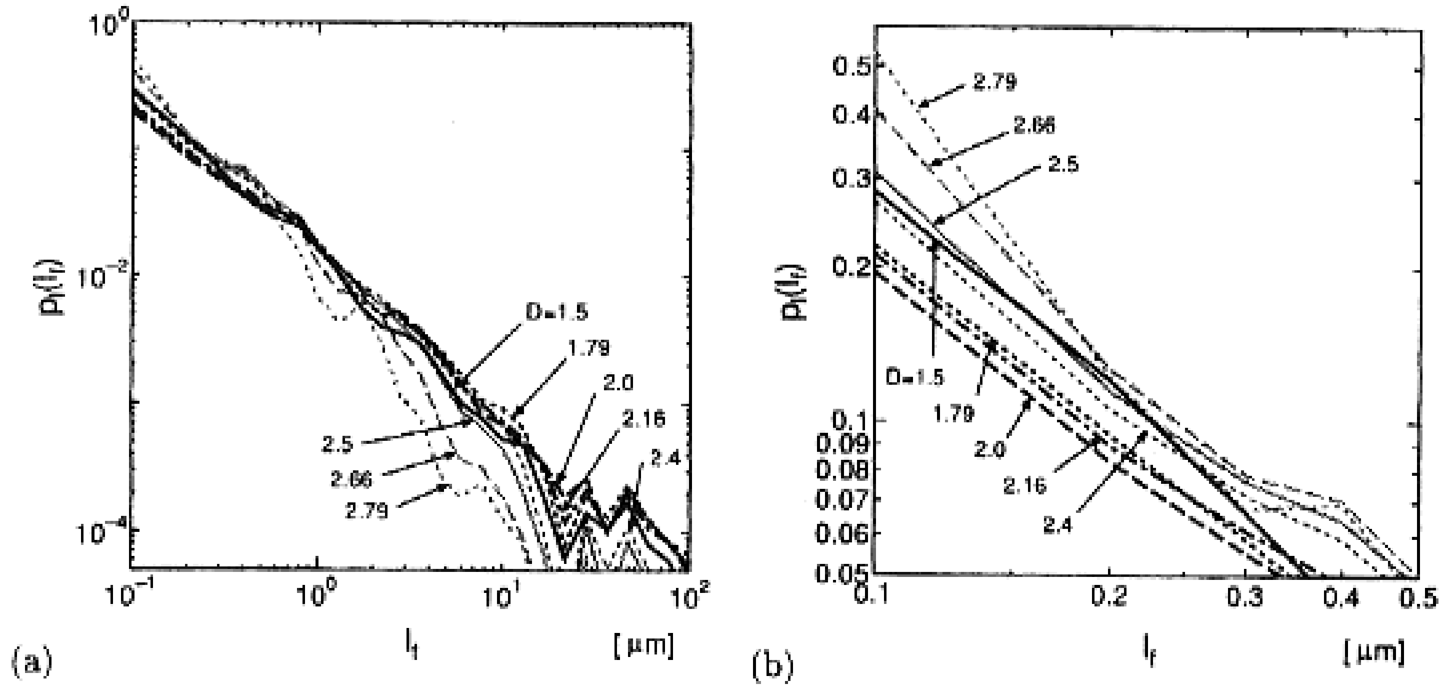

Figure 2. (a) Probability density functions of the scattering free path l_f calculated from the simulation data, and (b) their behavior for small l_f.

where $R = N_m/N_s$ is the multiple scattering efficiency. Therefore, the increase in the enhancement peak of the backscattered intensity indicates the relative increase in the multiple scattering. This is explained by the dimension dependence of the average volume density of the particles, which is given for the present media by

$$\rho_a = M_e^{D-3}, \tag{2}$$

as is the case of actual fractal aggregates of particles. It follows that the density increases with D, reaching unity at the limit of $D = 3$ corresponding to the closest cubic packing of particles. This increase in the particle density gives rise to the enhancement of R and, thus, raising the central intensity.

To make more visible the variation of the peak shape, Fig. 1(a) is normalized by $I_p(0)$ and shown in Fig. 1(b). It is seen that the slope of the shoulder of the peak changes depending on the fractal dimension D, increaseing with D up to $D = 2.5$ and, then, turning to decrease for $D > 2.5$. The former tendency agrees qualitatively with the previous simulation[5] and with the experimental results,[3,4] though the former simulation treated the case of $2.5 \leq D \leq 3.0$. A reason for the inverse dependence for $2.5 \leq D$ is discussed later.

3.2. PDFs of the scattering free path and the separation of end particle pairs

To examine how the object fractality affects the scattering process, the PDF $p_l(l_f)$ of the scattering free path l_f was calculated and is shown in Fig. 2. It is seen in this figure that, with an increase in D, probabilities of shorter paths decrease while those of longer paths decrease up to $D \approx 2$. This dependence is reversed, however, for larger D. This dependence for $D < 2$ can be explained from the spatial density distribution of particles. In case of mass fractals such as the Menger sponge and aggregated particles, the spatial density distribution $\rho(l)$ of particle, which gives the probability of finding a particle at distance l from any given particle in the medium, takes the form

$$\rho(l) \propto l^{D-3}. \tag{3}$$

For media with very small fractal dimensions, the average particle density is very low as eq. (2) implies and, therefore, it seems that the ray can take very long free path l_f. On the contrary, however, it is seen from eq. (3) that the probability of finding a next scattering particle at distance l decays rapidly with l and, consequently, probabilities of small l_f become relatively dominant. As D increases, probabilities of small l_f decrease as long as $D < 2$, where the medium is practically ***transparent***. When D exceeds 2, however, the medium enters the ***opaque*** regime and, in addition, becomes much more dense and, consequently, $p_l(l_f)$ for small l_f begins to increase with increasing D.

A quantity that affects directly the shape of the coherent peak is the probability of the separation δ of the first and last particles in the multiple scattering path, which in turn depends on D through l_f and the number of scattering n_s in a scattering path. The PDF $p_n(n_s)$ of n_s is shown in Fig. 3, while Fig. 4 shows the PDF $p_\delta(\delta)$ of δ. Figure 3 shows that probabilities of large n_s increase drastically with an increase in D because of the associated rapid increase in ρ_a. If $p_n(n_s)$ is independent of D, the D dependence of $p_\delta(\delta)$ would be similar to that of $p_l(l_f)$ shown in Fig. 2. However, the large increase in n_s with D has an effect of elongating δ, which will counteract the effect of reducing δ

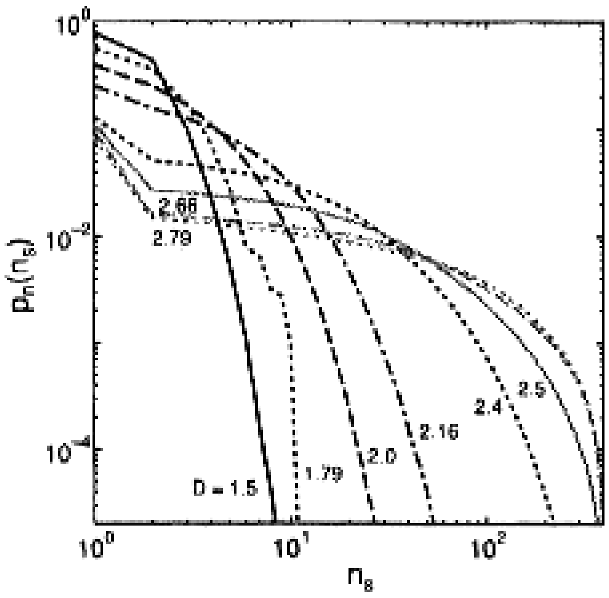

Figure 3. Probability density functions of the total number n_s of scattering from the first scattering to the last along each scattering path.

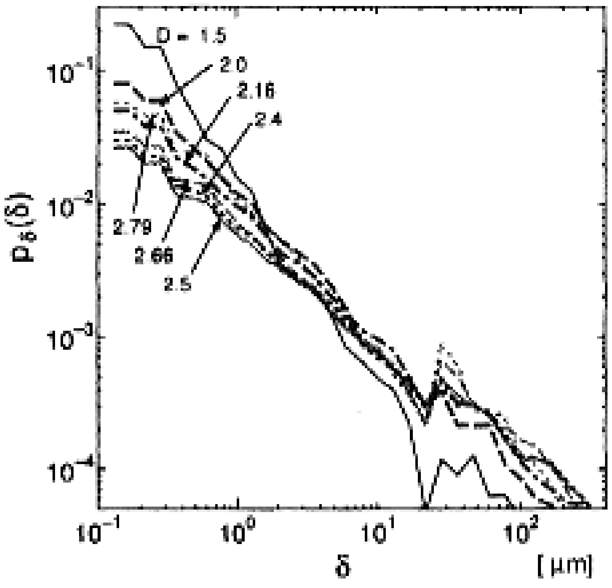

Figure 4. Probability density functions of the separation δ between the first and last scattering particles of each multiple scattering path.

due to the diminishing l_f for $D > 2$ and shift the turning point of the D dependence of $p_\delta(\delta)$ from $D = 2$ to a larger value. In fact, Fig. 4 shows that the turning point is shifted to $D = 2.5$. This explains why the D dependence of the slope in the coherent peak turns at $D \approx 2.5$.

In the simulation by Ishii et al.,[5] the slope of the coherent peak increases with D in the range of $2.5 < D < 3$, contradicting the present result. It is to be noted, however, that the present behavior of the coherent peak for $D > 2.5$ is mainly due to the high density of particles for extremely large D, where the particles are nearly in the closest packing and block almost completely the propagation of light. On the other hand, in the fractal media with a constant and moderate particle density used in Ref. [4], some rays can propagate far beyond neighboring particles and can *sense* the fractality of the media. In actual fractal aggregates, the particle density follows a power law similar to eq. (2), and increases drastically with D. However, it would be also true that the light can penetrate neighboring particles even in the dense and opaque region $D > 2$ when the particles have small scattering cross sections. Therefore, some intermediate dependence between the results of two simulations might be expected for actual fractal aggregates.

4. CONCLUSION

By means of a new approach on the basis of computer-generated fractal media, the D dependence of the coherent peak of the enhanced backscattering was demonstrated for the wide range of D. The shoulder of the coherent peak was shown to increase with D until $D = 2.5$, after which the tendency is inverted. This inverting anomaly was elucidated by extremely dense structures of the assumed media.

The simulation models of the previous[5] and present studies have some complementary features. The fractality is assumed indirectly through the PDF of l_f in the former and is assumed directly in media in the latter. The former is more theoretical while the latter is closer to actual experiments having scale limitations. Further combination of these approaches will be effective in better and more quantitative understanding of the multiple scattering phenomena in fractal media.

REFERENCES

1. Y. Kuga and A. Ishimaru, "Retroreflectance from a dense distribution of spherical particles," *J. Opt. Soc. Am. A* **1**(8), pp. 831–835, 1984.
2. E. Akkerman, P. E. Wolf, R. Maynard and G. Maret, "Theoretical study of the coherent backscattering of light by disordered media," *J. Phys. (Paris)* **49**, 77–98, 1988.
3. A. Dogariu, J. Uozumi and T. Asakura, "Enhancement of the backscattered intensity from fractal aggregates," *Waves in Random Media* **2**(4), pp. 259–263, 1992.
4. A. Dogariu, J. Uozumi and T. Asakura, "Enhancement factor in the light backscattered by fractal aggregated media," *Optcal Review* **3**(2), pp. 71–82, 1996.
5. K. Ishii, T. Iwai, J. Uozumi and T. Asakura, "Optical free-path-length distribution in a fractal aggregate and its effect on enhanced backscattering," *Appl. Opt.* **37**(21), 5014–5018, 1998.
6. B. Mandelbrot, *The Fractal Geometry of Nature*, Freeman, New York, 1983.