Influence of objects buried in highly dense media on angular correlations of the scattered intensity

S. Sangu,[a] T. Okamoto,[b] J. Uozumi[a] and T. Asakura[c]

[a]Research Institute for Electronic Science, Hokkaido University
Sapporo, Hokkaido 060-0812, Japan

[b]Department of Control Engineering and Science, Kyushu Institute of Technology
Iizuka, Fukuoka 820-8502, Japan

[c]Faculty of Engineering, Hokkai-Gakuen University, Sapporo, Hokkaido 064-0926, Japan

1. INTRODUCTION

We sometimes want to know some information hidden in highly dense media in which the light suffers a strongly multiple scattering, such as the existence of a tumor in a biological tissue and a state of a material behind a layer of paint. In such situations, optical measurement techniques which have the advantage of a non-contact and non-invasive method have been remarked and studied actively by many scientists.[1] In the present paper, we pay attention to speckle-like intensity fluctuations generated by multiply scattered light and to their angular correlation properties, and investigate an effect of objects buried in highly dense media on an angular correlation function of the scattered intensity. The angular correlation function is derived theoretically and numerical simulation results are shown.

2. ANGULAR INTENSITY CORRELATION FUNCTION: THEORY

For a random medium with a buried absorbing object, two scattering geometries of transmission and reflection are considered as shown in Fig. 1. Under an approximation that all scattering paths are mutually independent in the medium, the angular correlation function is given by

$$\begin{aligned} C(\Delta \boldsymbol{q}) &= [\langle I(\boldsymbol{q})I(\boldsymbol{q}+\Delta \boldsymbol{q})\rangle - \langle I\rangle^2]/\langle I\rangle^2 \\ &= \frac{1}{\langle I\rangle^2}\left|\frac{1}{A^2}\iint \langle I_i(\boldsymbol{\rho}')\rangle\langle|G(\boldsymbol{\rho},z;\boldsymbol{\rho}',z')|^2\rangle \exp(-i\Delta \boldsymbol{q}\cdot\boldsymbol{\rho}')d\boldsymbol{\rho}d\boldsymbol{\rho}'\right|^2, \end{aligned} \quad (1)$$

where $\Delta \boldsymbol{q}$ is a transversal component of wave vector with $\Delta q \approx k\Delta\theta$ for a small angular shift, $\Delta\theta$ being a difference between the scattering angles, and A, $\boldsymbol{\rho}$ and $\boldsymbol{\rho}'$ denote the illumination area, vector coordinates in the output plane and the illumination plane (the dotted lines in Fig. 1), respectively. $\langle I_i\rangle$ is an average intensity distribution in the illumination plane and is set to be zero in an area occupied by the object. G represents the Green function satisfying a wave equation and its mean square modulus $\langle|G|^2\rangle$ satisfies a diffusion equation under this approximation and has solutions for boundary conditions differently given for the transmission and reflection geometries. For the transmission geometry, $\langle|G|^2\rangle = 0$ at $z' = 0$ and L, where L is the sample thickness and the angular correlation is modified as

$$C(\Delta \boldsymbol{q}) \approx \left[\frac{\Delta qL}{\sinh(\Delta qL)}\right]^2 \left|\int \langle \bar{I}_i(\boldsymbol{\rho}')\rangle \exp(-i\Delta \boldsymbol{q}\cdot\boldsymbol{\rho}')\right|^2 d\boldsymbol{\rho}', \quad (2)$$

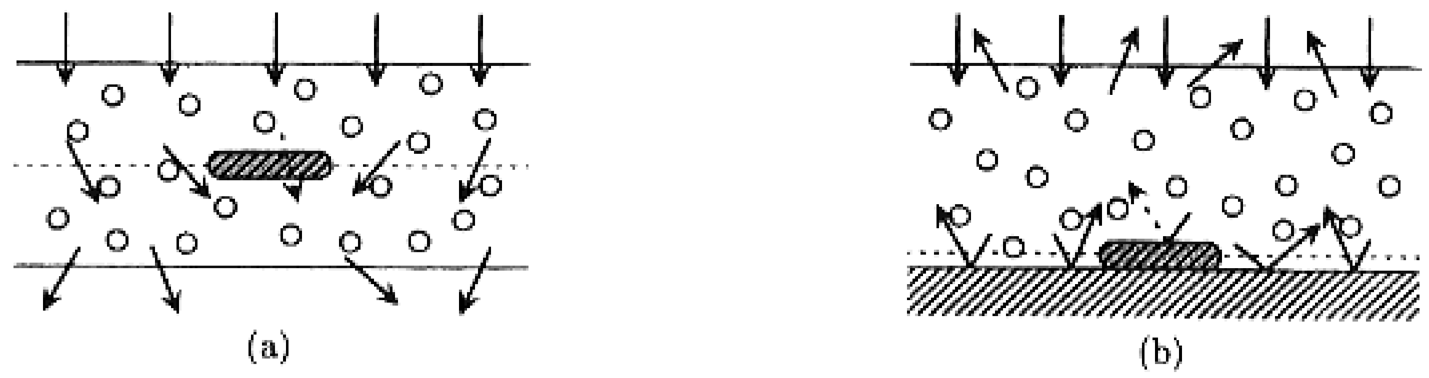

Figure 1. Scattering geometries for (a) transmitted and (b) reflected light.

Part of the 18th Congress of the International Commission for Optics: Optics for the Next Millennium
San Francisco, California • August 1999 SPIE Vol. 3749 • 0277-786X/99/$10.00

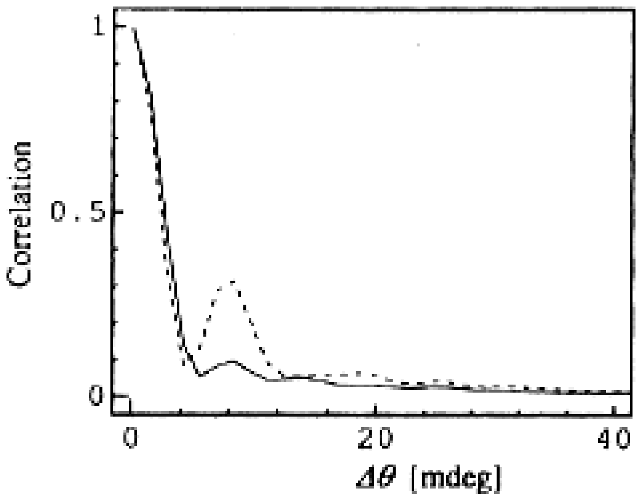

Figure 2. Angular correlation functions for the transmission geometry. The solid and dotted lines represent the cases with and without object, respectively.

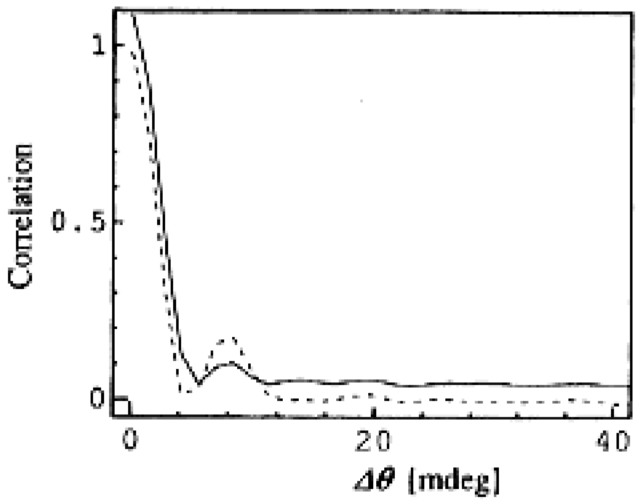

Figure 3. Angular correlation functions for the reflection geometry. The solid and dotted lines stand for $l^*/l = 1$ and 7.5, respectively.

where $\langle \overline{I}_i(\rho') \rangle$ is a normalized average intensity distribution in the illumination plane. For the reflection geometry, $\langle |G|^2 \rangle$ in Eq. (1) can be separated into two parts, $\langle |G_1|^2 \rangle$ and $\langle |G_0|^2 \rangle$, which are components reaching and not reaching the illumination plane, respectively. $\langle |G_0|^2 \rangle$ satisfies the same boundary condition as the transmission geometry and acts as a source term in $\langle |G_1|^2 \rangle$. $\langle |G_1|^2 \rangle$ satisfies the boundary condition $\langle |G_1|^2 \rangle = 0$ and $\partial \langle |G_1|^2 \rangle / \partial z' = 0$ at $z' = 0$ and L, respectively, and thus, the angular correlation function is finally given by

$$C(\Delta q) \propto \left[1 - \frac{\Delta q l^* \cosh(\Delta q L)}{\sinh(\Delta q L)} \right]^2 \left| \frac{1}{A} \int \exp(-i\Delta \boldsymbol{q} \cdot \boldsymbol{\rho}) d\boldsymbol{\rho} + \frac{D}{\cosh(\Delta q L)} \int \langle \overline{I}_i(\boldsymbol{\rho}') \rangle \exp(-i\Delta \boldsymbol{q} \cdot \boldsymbol{\rho}') d\boldsymbol{\rho}' \right|^2, \quad (3)$$

where D is the constant depending on a sample dimensionality. Both of Eqs. (2) and (3) involve the Fourier transform of the intensity distribution in the illumination plane, and it is found that the presence of the object affects the angular correlation function of the scattered light. For the reflection geometry, however, the degree of the effect is strongly dependent on the transport mean free path l^* of the medium because the scattering component not reaching the illumination plane also contributes to the angular correlation function.

3. SIMULATION

The angular correlation function is evaluated by numerical simulations based on matrix calculation of the complex amplitude.[2,3] In Fig. 2, the angular correlation functions are plotted in the cases with and without the object, where the object size, sample thickness, and sample width are set to be $w = 32l$, $L = 10l$, and $W = 128l$, respectively, and a random field is injected in the illumination plane. The correlation peaks at $\Delta\theta = 0$ represent the average speckle size, while the height of the second peak at $\Delta\theta \approx 8$ mdeg depends on the object size and the illumination area. These properties of the correlation peak correspond to the prediction of the Fourier transform relation in Eq. (2).

Figure 3 shows the angular correlation functions for the reflection geometry. The object size is fixed at $w = 32l$, and the transport mean free path is set at $l^*/l = 1$ and 7.5. The correlation peak appears at the same angular shift as the transmission geometry, but is quite small for $l^*/l = 1$ because the scattering component not reaching the illumination plane is dominant in the strong scattering regime of $l^*/l = 1$.

From these results, intensity fluctuations generated by multiply scattered light may be useful for detecting the object buried in highly dense media.

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