Bispectrum Phase Estimation For $\pi/4$ DQPSK Signals Under Flat Fading

Kunio Takaya and David E. Dodds†
Electrical Engineering, University of Saskatchewan
57 Campus Drive, Saskatoon, Sask. S7N 5A9 , Canada
Phone: (306) 966-5392, Fax: (306) 966-5407, email: kunio_takaya@engr.usask.ca
†Telecommunications Research Laboratories (TR*Labs*)
108-15 Innovation Blvd., Saskatoon, Sask. S7N 2X8 , Canada
Phone: (306) 668-9302, Fax: (306), 668-1944 email: dodds@sask.trlabs.ca

Abstract

When $\pi/4$ offset DQPSK signals are subjected to time varying multi-path reception, both amplitude and phase of the signal experience severe distortion. This effect is called flat (in frequency) fading, which causes sudden signal weakening often observed in a moving vehicle. AGC (Automatic Gain Control) can compensate amplitude fluctuations since the modulus of $\pi/4$ offset DQPSK is constant. On the other hand, phase fluctuation must be corrected based on the knowledge of amplitude fluctuation. It is known that instantaneous phase differential can be estimated from the amplitude, provided if signal is of minimum (or maximum) phase. Unfortunately, any segment of $\pi/4$ offset DQPSK signals rarely satisfies this minimum phase condition because of the abrupt phase changes that occur in a multiple of $\pi/4$. The third order cummulant, or its Fourier transform called bispectrum, is capable of detecting the phase of signals regardless of whether they are minimum phase or non-minimum phase. This paper discusses how to estimate the phase when flat fading obscures the phase of $\pi/4$ offset DQPSK carrier signal.

I. Introduction

The emerging PCS standard IS-54 has adopted $\pi/4$ offset DQPSK digital modulation scheme for wireless cellular systems. The $\pi/4$ offset DQPSK is phase modulation of a constant modulus. Mobile RF channels experience short term fading due to the standing wave established by the superposition of waves travelling over multiple paths of different lengths. The nulls of the standing wave occur every half wavelength of the RF wave. It has been reported that a vehicle travelling a velocity of 100km/h will encounter a null at every 6.3ms if RF frequency is at 850MHz. [2] The $\pi/4$ offset DQPSK operating at a sub-carrier frequency is affected by this flat fading of RF propagation channel. Flat fading implies that all frequencies receives the same amount of fading at any time. A $\pi/4$ offset DQPSK signal can be represented by

$$s_0(t) = A_0\, e^{j[\omega t + \frac{\pi}{4} n(t)]}. \tag{1}$$

$s_0(t)$ is a complex wave to be usually observed in the I-Q phase plane. Where, $n(t)$ is one of the inter number $n = 0, 1, 2,$ or 3 representing a DQPSK code. When this signal is transmitted through a flat fading channel, it receives amplitude and phase distortion. The received and demodulated signal in the base band is, therefore,

$$s(t) = A(t)\, e^{j\phi(t)}. \tag{2}$$

Where, $A(t)$ is amplitude fluctuation and $\phi(t)$ represents phase fluctuation. Disregard the phase modulation $\frac{\pi}{4}n(t)$ for the purpose of compensating the phase variation caused by the flat fading. Though practical RF channels are not of a flat or a raised cosine frequency response which is free from inter-symbol interference (ISI), it is assumed that the channel itself is ideal. Since $\pi/4$ offset DQPSK does not depend on the amplitude, $A(t)$ is less important. The phase $\phi(t)$, however, severely affects the detection of $\frac{\pi}{4}$ phase modulation. Automatic gain control (AGC) can keep $A(t)$ reasonably constant. It is apparent that the problem is to estimate $\phi(t)$. In order to separate the phase, we now consider $\ln s(t)$ instead of $s(t)$.

$$\frac{d}{dt}\ln s(t) = \frac{1}{A(t)}\frac{d}{dt}A(t) + j\frac{d}{dt}\phi(t). \tag{3}$$

Thus, the problem of estimating the phase $\phi(t)$ now becomes a problem of finding the imaginary part $j\frac{d}{dt}\phi(t)$ from the real part $\frac{1}{A(t)}\frac{d}{dt}A(t)$. A feasible solution exists and is presented in the article [2] as

$$\frac{d\phi(t)}{dt} = H\left[\frac{1}{A(t)}\frac{dA(t)}{dt}\right], \tag{4}$$

where H is the Hilbert transform. A different and more detailed interpretation for this solution is given in the next section. This solution works for such signals that satisfy the minimum or maximum phase condition. It is necessary to develop a more robust method that always work regardless of signals property. It is known that the higher order power spectrum, which reflects the statistical higher order cummulant, restores the phase information. Therefore, bispectrum (the next higher to power spectrum) is considered in this paper to exploit its properties for phase estimation.

II. Recovery of Real or Imaginary Part of a Complex Signal

Suppose we have a complex signal $s(n) = x(n) + j\,y(n)$, for which $s(n)$ is defined only for half a range of $n = 0, 1, \cdots, 2N-1$, i.e. $n = 0, 1, \cdots, N-1$ and zero for $n =$

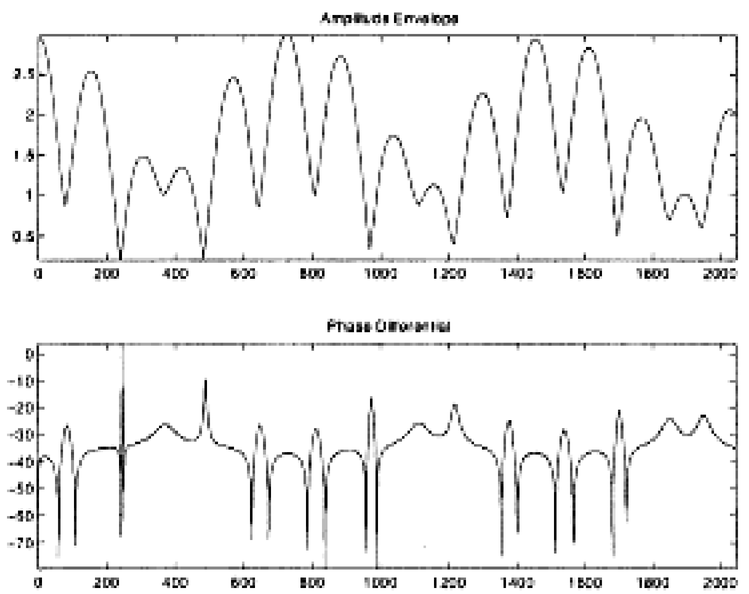

Figure 1: Amplitude and phase differential changes over a course of time. (Phase in dB)

$N, N+1, \cdots, 2N-1$. Under this constraint, the real part and the imaginary part of $s(n)$ are not independent and are mutually related. Either part is the Hilbert transform of the other part. In order to show this relationship, we first introduce an even symmetic wave $x_e(n)$ and an odd symmetic wave $y_o(n)$. Assuming $y(N)=0$,

$$x_e(n) = \begin{cases} x(n) & \text{if } n = 0, 1, \cdots, N-1 \\ x(2N-n) & \text{if } n = N, \cdots, 2N-1 \end{cases} \quad (5)$$

$$y_o(n) = \begin{cases} y(n) & \text{if } n = 0, 1, \cdots, N-1 \\ -y(2N-n) & \text{if } n = N, \cdots, 2N-1 \end{cases} \quad (6)$$

We next consider the discrete Fourier transform of $x_e(n)$ and $y_o(n)$ indivisually.

$$X(k) = \mathcal{DFT}\{x_e(n)\} \qquad Y(k) = \mathcal{DFT}\{y_o(n)\} \quad (7)$$

Since $x_e(n)$ is a even sequence, $X(k)$ is real and symmetric. Similarly, $Y(k)$ is imaginary and anti-symmetric. Now we force $X(k)$ to become anti-symmetric and $Y(k)$ to become symmetric with respect to their non-zero part, either real or imaginary part, introducing $X_o(k)$ and $Y_e(k)$.

$$X_o(k) = \begin{cases} X(n) & \text{if } n = 0, 1, \cdots, N-1 \\ -X(2N-n) & \text{if } n = N, \cdots, 2N-1 \end{cases} \quad (8)$$

$$Y_e(n) = \begin{cases} Y(n) & \text{if } n = 0, 1, \cdots, N-1 \\ Y(2N-n) & \text{if } n = N, \cdots, 2N-1 \end{cases} \quad (9)$$

Letting,

$$x_o(n) = \mathcal{IDFT}\{X_o(k)\} \quad y_e(n) = \mathcal{IDFT}\{Y_e(k)\} \quad (10)$$

we now have two time sequences, $x_o(n)$ and $y_e(n)$. Due to the fact that $X_o(k)$ is real and symmetric, $x_o(n)$ is a imaginary and symmetric sequence, whereas $y_e(n)$ is a real and anti-symmetric sequece since $Y_e(k)$ is imaginary and symmetric. A composite complex signal constructed by combining the real part and the imaginary part can be expressed by $y_e(n) + j\,x_o(n)$. Comparing this to the introduced two sided signal $x_e(n) + j\,y_o(n)$, In order to construct the one sided signal from these two sided signals,

$$x(n)+j\,y(n) = \frac{1}{2}\{x_e(n)+y_e(n)\}+j\,\{y_o(n)+x_o(n)\} \quad (11)$$

the necessary conditions are

$$x_e(n) = y_e(n) \text{ and } y_o(n) = x_o(n) \quad (12)$$

for $n = 0, 1, \cdots, N-1$. Or in the Fourier domain,

$$X(k) = -j\,Y_e(k) \text{ and } Y(k) = j\,X_o(k). \quad (13)$$

Thus, we can obtain the real part $x(n)$ from the imaginary part $y(n)$ or vice versa. This process of reproducing the original signal $x(n)+j\,y(n)$ from its two sided counter part $x_e(n)+j\,y_o(n)$ is equivalent to applying the Hilbert tranform,

$$H(k) = \begin{cases} -j & \text{if } k = 0, 1, \cdots, N-1 \\ j & \text{if } k = N, \cdots, 2N-1 \end{cases} \quad (14)$$

to the IDFT of $X_o(k)+j\,Y_e(k)$. Provided that the original complex signal is one sided in the similar sense of single side band spectrum, the real part and imaginary part are related by the Hilbert transform.

III. Flat Fading Channels

A $\pi/4$ offset DQPSK signal received in a moving vehicle affected by multi-path propagation is generally expressed by the following equation.

$$s(t) = \sum_i c_i\, e^{j\{\omega t + \frac{\pi}{4} n(t) + \alpha_i \sqrt{\beta_i t^2 + \gamma_i}\}} \quad (15)$$

The summation over the subscript i means to account all paths of propagation. c_i represents the attenuation in path i. α_i, β_i and γ_i are the constants that depend on the geometry of surrounding reflection surfaces. $n(t)$ is an integer representing a symbol being transmitted. A case of strong reflections from three near-by surfaces is shown in Fig. 1. The amplitude shows dramatic changes that accompany changes in the phase differential (first time derivative of the phase). This information of phase differential can only be calculated by the signal model given in Eq. 15 but not available for practical receivers. If the received signal affected by multi-path propagation is compensated only in terms of amplitude, the constellation resulted from this $\pi/4$ offset DQPSK becomes as shown in Fig. 2. No distinct four positions of stars are visible.

IV. Bispectrum Phase Estimation

The determination of phase from known power spectrum is possible only if a signal is definitely of minimum phase. In order to estimate phase from the magnitude of a signal, the higher order spectrum, at least the bispectrum, must be used instead of its power spectrum. [5] When the auto-correlation function $r(\tau)$ of a signal is known, the power spectrum is given by

$$S(j\omega) = \int_{-\infty}^{\infty} r(\tau) e^{-j\omega\tau} d\tau. \quad (16)$$

Statistically, the auto-correlation function $r(\tau)$ is given by

$$r(\tau) = E[x(t)x(t+\tau)] \quad (17)$$

for a stationary signal $x(t)$. Extending this to the third order cummulant of the signal,

$$c(\tau_1, \tau_2) = E[x(t)x(t+\tau_1)x(t+\tau_2)] \quad (18)$$

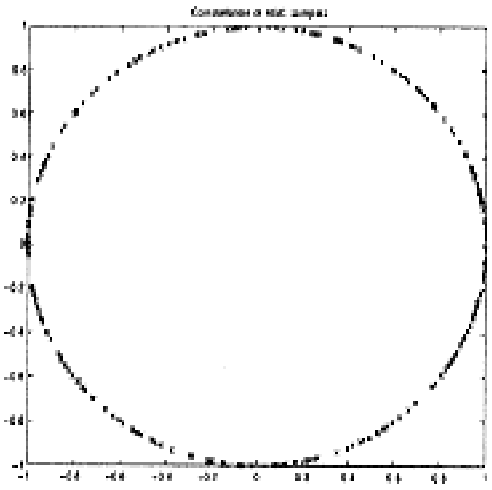

Figure 2: The constellation of AGC compensated decoded data.

the bispectrum is obtained from the Fourier transform,

$$B(j\omega_1, j\omega_2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} c(\tau_1, \tau_2) \times e^{-j(\omega_1\tau_1+\omega_2\tau_2)} d\tau_1\, d\tau_2. \quad (19)$$

For discretized time t, the third order cummulant of the stationary signal $x(t)$, Eq. 18 can be written as,

$$c(\tau_1, \tau_2) = \sum_{t=-\infty}^{\infty} x(t)x(t+\tau_1)x(t+\tau_2). \quad (20)$$

The discrete form of the Fourier transform given in Eq. 19 is given

$$\begin{aligned} &B(j\omega_1, j\omega_2) \\ &= \gamma \sum_{-\infty}^{\infty}\sum_{-\infty}^{\infty} c(\tau_1, \tau_2)e^{-j(\omega_1\tau_1+\omega_2\tau_2)} \\ &= \gamma \sum_{\tau_1}\sum_{\tau_2}\sum_{t} x(t)x(t+\tau_1)x(t+\tau_2)e^{-j(\omega_1\tau_1+\omega_2\tau_2)} \\ &= \gamma X(j\omega_1)X(j\omega_2)X(-j(\omega_1+\omega_2)) \\ &= \gamma X(j\omega_1)X(j\omega_2)X^*(j(\omega_1+\omega_2)) \quad (21) \end{aligned}$$

Where, γ is a constant of proportionality and $X(j\omega)$ is the Fourier transform defined on discretized time,

$$X(j\omega) = \sum_{t=-\infty}^{\infty} x(t)e^{-j\omega t} = |X(j\omega)|e^{j\phi(\omega)}. \quad (22)$$

Since the bispectrum $B(j\omega_1, j\omega_2)$ is decomposed into the magnitude and the phase,

$$B(j\omega_1, j\omega_2) = |B(j\omega_1, j\omega_2)|e^{j\varphi(\omega_1,\omega_2)}. \quad (23)$$

Thus, the bispectrum phase and the phase of the signal $x(t)$ are related by

$$\varphi(\omega_1, \omega_2) = \phi(\omega_1) + \phi(\omega_2) - \phi(\omega_1+\omega_2). \quad (24)$$

Our problem here is to estimate the phase $\phi(\omega)$ in Eq. 22 for a given signal $x(t)$. The Fourier transform $X(j\omega)$ is not available because of the direct consequence of the problem set up. However, the bispectrum $B(j\omega_1, j\omega_2)$ is available as a result of Eq. 19, which is simply the 2-dimensional Fourier transform of the third order cummulant $c(\tau_1, \tau_2)$. In order to find $\phi(\omega)$ from $B(j\omega_1, j\omega_2)$, the basic phase relationship of Eq. 24 is exploited. A number of algorithms have been developed in this regard. Most algorithms estimate the phase by minimizing the mean square error MSE defined as

$$\begin{aligned}\text{MSE}= \int_{-\pi}^{\pi}\int_{-\pi}^{\pi} &|\varphi(\omega_1, \omega_2) - \hat\phi(\omega_1) \\ &-\hat\phi(\omega_2) + \hat\phi(\omega_1+\omega_2)|^2 d\omega_1 d\omega_2. \quad (25)\end{aligned}$$

When the signal $x(t)$ is descritized in time, both $\varphi(\omega_1, \omega_2)$ and $\phi(\omega)$ are 2π periodic. Then, both the bispectrum phase and the signal phase can be expressed in the Fourier series,

$$\begin{aligned} c_{n_1,n_2} &= \frac{1}{(2\pi)^2}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi} \varphi(\omega_1, \omega_2) \\ &\times\ e^{-jn_1\omega_1}e^{-jn_2\omega_2} d\omega_1 d\omega_2 \quad (26)\end{aligned}$$

$$c_n = \frac{1}{(2\pi)}\int_{-\pi}^{\pi} \phi(\omega)e^{-jn\omega}d\omega \quad (27)$$

The Fourier series coefficients of the optimum phase which minimizes MSE can be found to be [4],

$$\begin{aligned} \hat c_0 &= -c_{0,0} \\ \hat c_n &= \frac{1}{3}(c_{n,0} + c_{0,n} - c_{n,n}). \quad (28)\end{aligned}$$

The phase estimate is thus obtained by

$$\phi(\omega) = \sum_{n=-\infty}^{\infty} \hat c_n e^{jn\omega}. \quad (29)$$

For example, a discrete signal given by

$$\begin{aligned} x(k) = e^{-4\frac{k}{N}}[&\cos(8\frac{2\pi k}{N}) \\ &+ \cos(16\frac{2\pi k}{N}) + e^{4.5\frac{k}{N}}] \quad (30)\end{aligned}$$

exhibits non-minimum phase characteristics having some zeros outside of the unit circle in the z-plane. The phase was estimated from its amplitude by using the bispectrum. The result shown in Fig. 3 shows a good agreement between the known true phase and the estimated phase.

V. Implementation of Bispectrum Phase Estimation for $\pi/4$ DQPSK

As discussed in the previous section, the phase that can be estimated by the bispectrum is the phase defined in frequency domain. The phase that needs to be determined in $\pi/4$ DQPSK signals is the instantaneous phase $\phi(t)$ of Eq. 2. This instantaneous phase $\phi(t)$ can only be found as a phase of $\pi/4$ DQPSK carrier frequency in the frequency response of the channel. Since the frequency shift due to phase $\phi(t)$ is practically much smaller than the carrier frequency ω, a short time observation of $\pi/4$ DQPSK signal which covers a certain number of carrier cycles can be used to calculate the frequency response. This observed signal constitutes the output in the calculation of the impulse response. As for the input, the

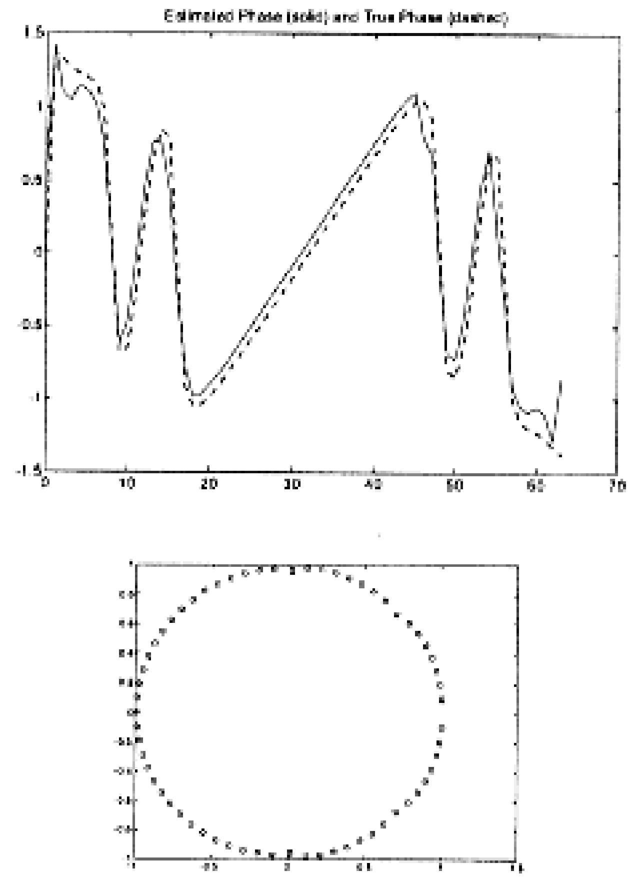

Figure 3: Comparison between the true phase (dashed line) and the estimated phase (solid line) of a non-minimum phase signal shown by the zero locations

constant modulus AGC compensated signal is the closest candidate for the unknown input. The transmitted signal unaffected by multi-path propagation is unknown. However, it is reasonable to assume that the combined effects of multi-path propagation is mostly reflected to the amplitude change and the carrier waveform has received little change. Mathematically, the input signal to be used in the impulse response determination is,

$$u(t) = \sum_i c_i \, e^{\, j(\omega t + \frac{\pi}{4} n(t))}. \tag{31}$$

which is nothing but the constant modulus AGC compensated output waveform. This pair of input and output often results in a non-minimum phase impulse response, thus requiring bispectrum analysis to obtain the true phase $\phi(t)$.

This limitation can be however loosened by introducing another assumption that unmodulated carrier signal will also result in the same amplitude (envelope) change. The input and output pair under this assumption is,

$$u(t) = \sum_i c_i \, e^{\, j\omega t} \tag{32}$$

$$s(t) = \sum_i c_i \, e^{\, j(\omega t + \alpha_i \sqrt{\beta_i t^2 + \gamma_i})} \tag{33}$$

Since the output contains frequency components not contained in the single frequency input (non-linear system), obvious singularities make it inevitable to ignore all frequencies other than the carrier frequency in calculating the frequency response $G(j\omega)$. This assumption leads to an approximated phase angle θ and its corresponding

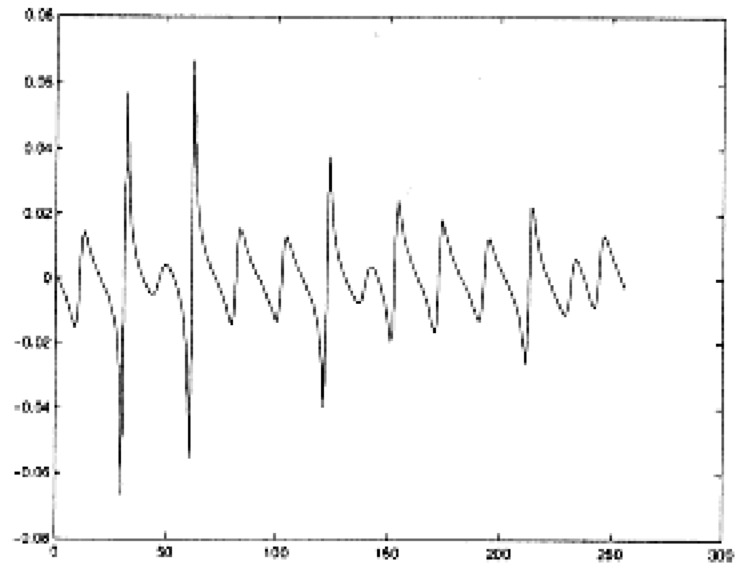

Figure 4: Calculated phase from the amplitude fluctuation shown in Fig. 1.

lead/lag time T.

$$G(j\omega) = \frac{S(j\omega)}{U(j\omega)} = A\, e^{j\theta} = A\, e^{j\omega T}. \tag{34}$$

The impulse response thus obtained tends to be of minimum phase, justifying the method to estimate phase differential from amplitude differential given by Eq. 4. The phase calculated with this second assumption is shown in Fig. 4. Comparing the phase differential shown in Fig. 1 and Fig. 4, it is evident that the time derivative of Fig. 4 is the phase differential shown in in Fig. 1.

VI. Conclusion

The direct use of a few cycles of amplitude varying $\pi/4$ DQPSK signal for bispectrum analysis has an advantage that the phase modulation introduces a wide range of frequency components. The abrupt $\pi/4$ phase changes, i.e. discontinuities, gives rise to a widely spread spectrum. However, they contribute little to the frequencies near the carrier frequency ω, which are needed to determine more accurate phase $\phi(t)$ of the specific carrier frequency ω. By experience, the second assumption works better if the single frequency ω is perturbed around the carrier frequency to find the slope of the linear phase response, in another word the average time lead/lag. In this case, the channel model is definitely of minimum phase which allows the use of ordinary spectrum instead of bispectrum. The bispectrum approach, of course, works as well. Another finding observed is that a small number of carrier cycles such as one cycle is actually better in terms of tracking ability for fast phase changes. The amount of calculations involved is dramatically reduced in such a small number of carrier cycles, particularly for the bispectrum implementation. Note that the number of samples per carrier cycle used in the experiments shown in this paper is eight sample per cycle.

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