Carrier Phase Recovery For Constant Modulus QAM Signals Under Flat Fading

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ABSTRACT

When $\pi/4$ offset DQPSK signals are subjected to time varying multi-path reception, both amplitude and phase of the signal experience severe distortion. This effect is called flat (in frequency) fading, which causes sudden signal weakening often observed in a moving vehicle. AGC (Automatic Gain Control) can compensate amplitude fluctuation since the modulus of $\pi/4$ offset DQPSK is constant. On the other hand, phase fluctuation must be corrected based on the knowledge of amplitude fluctuation. It is known that instantaneous phase differential can be estimated from the amplitude, provided if signal is of minimum (or maximum) phase. This paper discusses how to estimate the phase when flat fading obscures the phase of constant modulus QAM carrier signal by using the Hilbert transform and the amplitude information.

I. Introduction

The emerging PCS standard IS-54 has adopted $\pi/4$ offset DQPSK digital modulation scheme for wireless cellular systems. The $\pi/4$ offset DQPSK is phase modulation of a constant modulus. Mobile RF channels experience short term fading due to the standing wave established by the superposition of waves travelling over multiple paths of different lengths. The nulls of the standing wave occur every half wavelength of the RF wave. It has been reported that a vehicle travelling a velocity of 100km/h will encounter a null at every 6.3ms if RF frequency is at 850MHz. [2] The $\pi/4$ offset DQPSK operating at a sub-carrier frequency is affected by this flat fading of RF propagation channel. Flat fading implies that all frequencies receives the same amount of fading at any time. A constant modulus QAM signal of the Nth order can be represented by

$$s_0(t) = A_0\, e^{j[\omega t + \frac{2\pi}{N} n(t) + \theta]}. \qquad (1)$$

$s_0(t)$ is a complex wave to be usually observed in the I-Q phase plane. Where, $n(t)$ is one of the integer number $n = 0, 1, 2,$ or $N-1$ representing a symbol of N possible symbols for the constant modulus QAM. θ is an offset angle. In the case of $\pi/4$ offset DQPSK, $N = 4$ and $\theta = \pi/4$. When this signal is transmitted through a flat fading channel, it receives amplitude and phase distortion. The received and demodulated signal in the base band is, therefore,

$$s(t) = A(t)\, e^{j\phi(t)}. \qquad (2)$$

Where, $A(t)$ is amplitude fluctuation and $\phi(t)$ includes the phase shift by encoding and phase fluctuation. In the problem of estimating the carrier phase, the phase fluctuation due to multi-path propagation and the Doppler effect needs to be estimated. If the carrier phase fluctuation is estimated entirely from the magnitude, and only the special case of the constant modulus QAM is considered, the phase shift introduced by QAM modulation can be ignored, since the QAM modulation does not affect the amplitude $A(t)$. The phase fluctuation, now represented by $\phi(t)$ severely affects the detection of the modulated phase of $\frac{2\pi}{N}n(t)$. Automatic gain control (AGC) can keep $A(t)$ reasonably constant so that the key to the problem is to estimate $\phi(t)$. Practical RF channels are not flat over the frequency range to $f_s/2$ (half a sampling frequency) or a raised cosine frequency response which warrants the channel to be free from inter-symbol interference (ISI). However, it is assumed here that the channel itself is ideal.

In order to separate the phase, we now consider $\ln s(t)$ instead of $s(t)$ and differentiate it with respect to time t.

$$\frac{d}{dt}\ln s(t) = \frac{1}{A(t)}\frac{d}{dt}A(t) + j\frac{d}{dt}\phi(t). \qquad (3)$$

Thus, the problem of estimating the phase $\phi(t)$ now becomes a problem of finding the imaginary part $j\frac{d}{dt}\phi(t)$ from the real part $\frac{1}{A(t)}\frac{d}{dt}A(t)$. There is a feasible solution to the problem of finding the real part from the imaginary part or vice versa, presented in the articles [1][5]. A similar work applied to the specific case of $\pi/4$ offset DQPSK problem estimating the phase differential from the amplitude is reported in the article [2].

$$\frac{d\phi(t)}{dt} = \mathcal{H}\left[\frac{1}{A(t)}\frac{dA(t)}{dt}\right], \qquad (4)$$

where $\mathcal{H}$ is the Hilbert transform. A more detailed interpretation for this solution is given in the next section. It is known that this solution requires signals to satisfy the minimum or maximum phase condition.

II. Recovery of Real or Imaginary Part of a Complex Signal

Suppose we have a complex signal $s(n) = x(n) + j\,y(n)$, for which $s(n)$ is defined for the range of $n = 0, 1, \cdots, N-1$. Define its mirror image sequence x_m and $y_m(n)$ such that

$$x_m(n) = x(N-n-1), \quad y_m(n) = y(N-n-1). \quad (5)$$

We first augment the signal $x + jy$ with $x_m(n)$ and $y_m(n)$ to double the length to make it twice as long.

$$(x) + j(y) \Longrightarrow s_1 = (x, x_m) + j(y, -y_m) \quad (6)$$

The resulting vector (x, x_m) is apparently symmetric and $(y, -y_m)$ is asymmetric. Define the operator $\mathcal{ASYM}$ that transforms a symmetric vector to an asymmetric vector to make the subsequent discussion simple. For example, $\mathcal{ASYM}(x, x_m) = (x, -x_m)$. Also, $\mathcal{DFT}$ and $\mathcal{IDFT}$ are the discrete Fourier transform and the inverse discrete Fourier transform operator, respectively. Consider the following operations:

$$\begin{aligned} &\mathcal{IDFT}\{\mathcal{ASYM}\{\mathcal{DFT}(x, x_m)\}\} \\ &\quad = (0,0) + j(\tilde{x}, \tilde{x}_m) \quad (7) \\ &\mathcal{IDFT}\{\mathcal{ASYM}\{\mathcal{DFT}(y, y_m)\}\} \\ &\quad = (\tilde{y}, -\tilde{y}_m) + j(0,0) \quad (8) \end{aligned}$$

Since (x, x_m) is symmetric, its DFT results in a real symmetric vector. Converting the symmetric DFT into an asymmetric vector, the last operator of IDFT produces an symmetric imaginary vector. Similarly, the DFT applied to the asymmetric $(y, -y_m)$ results in an imaginary asymmetric vector. Forcing this to become a symmetric vector by the asymmetric operator, the last operator of IDFT produces an real asymmetric vector. Since the result of Eq. 7 is imaginary and the result of Eq. 8 is real, these are now combined to make a new vector,

$$s_2 = (\tilde{y}, -\tilde{y}_m) + j(\tilde{x}, \tilde{x}_m) \quad (9)$$

We now seek for a condition to reconstruct a zero padded original signal s having the size of $2N$ from the two signals s_1 and s_2. That is,

$$\begin{aligned} &(x, 0) + j(y, 0) \\ &= \frac{1}{2}[\, s_1 + s_2 \,] \\ &= \frac{1}{2}[(x, x_m) + j(y, -y_m)] + \frac{1}{2}[(\tilde{y}, -\tilde{y}_m) + j(\tilde{x}, \tilde{x}_m)] \\ &= \frac{1}{2}[(x + \tilde{y}, x_m - \tilde{y}_m) + j(y + \tilde{x}, -y_m + \tilde{x}_m)] \quad (10) \end{aligned}$$

Therefore, we find the necessary conditions, $x_m = \tilde{y}_m$ and $\tilde{x}_m = y_m$. This implies that $x = \tilde{y}$ and $\tilde{x} = y$, because x_m and y_m are the mirror images of x and y. Recalling the fact that the three stage operations of $\mathcal{IDFT}\{\mathcal{ASYM}\{\mathcal{DFT}(.,.)\}\}$ followed by swapping the real part and the imaginary part is equivalent to the discrete Hilbert transform $\mathcal{H}\{.\}$, we have $\tilde{x} = \mathcal{H}\{x\}$ and

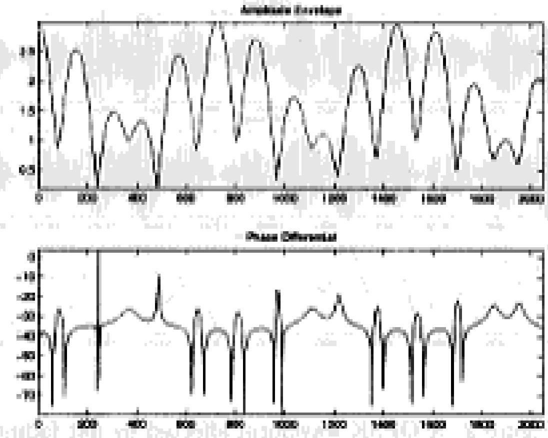

Figure 1: Amplitude and phase differential changes over a course of time. (Phase in logarithmic scale)

$\tilde{y} = \mathcal{H}\{y\}$. Thus, we can conclude with the final result,

$$x = \tilde{y} = \mathcal{H}\{y\} \quad (11)$$

$$y = \tilde{x} = \mathcal{H}\{x\}. \quad (12)$$

The real part and the imaginary part of the signal $s(n) = x(n) + jy(n)$ are related by the Hilbert transform. However, so called minimum phase condition must be met for the above relationship to hold. If a zero of the signal sequence $s(n)$, $n = 0, 1, \cdots, N-1$ is located at $Re^{j\theta}$, the contribution of its reciprocal zero $R^{-1}e^{-j\theta}$ is equal in magnitude, but different in phase. Therefore, it is known that phase cannot be uniquely determined from magnitude, unless the minimum phase condition is imposed to the signal sequence $s(n)$. Finding the phase from the magnitude in the polar coordinates, or the imaginary part from the real part in the cartesian coordinates is possible only when all zeros of the signal sequence is confined within the unit circle, i.e. minimum phase. This condition causes problems when the method is implemented in a practical problem.

III. A Model of Flat Fading Channels

A model of the constant modulus signal encoded with N code words, received in a moving vehicle, affected by multi-path propagation, can be represented by the following equation.

$$s(t) = \sum_i c_i\, e^{\,j(\omega t + \frac{2\pi}{N} n(t) + \alpha_i \sqrt{\beta_i t^2 + \gamma_i} + \zeta_i)} \quad (13)$$

The summation over the subscript i means to account all paths of propagation. c_i represents the gain in path i. α_i, β_i, γ_i and ζ_i are the constants that depend on the the locations of the base station, reflection surfaces and the location and velocity of the vehicle. $n(t)$ is an integer representing a symbol being transmitted. A case in which reflections come from three near-by surfaces is shown in Fig. 1. The amplitude shows dramatic changes that accompany changes in the phase differential (first time derivative of the phase). This information of phase differential can only be calculated by the signal model given in Eq. 13 but not available for practical receivers.

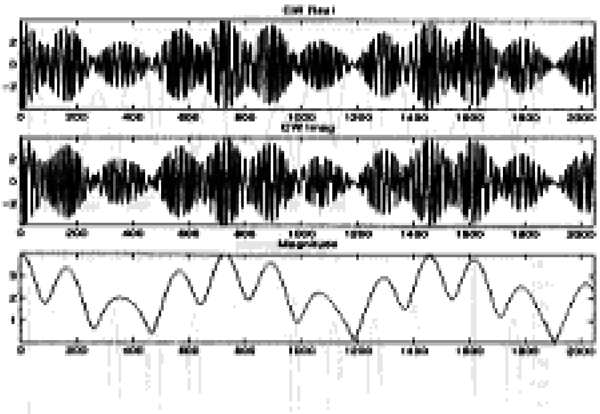

Figure 2: A QPSK waveform affected by flat fading

Large spikes can be observed in the graph of phase deferential when the amplitude encounters the maxima or the minima. It is also apparent that the time derivative of the amplitude is reflected to the phase differential, supporting the basic relationship given by Eq. 4. Received constant modulus QAM signals affected by multi-path propagation is compensated in terms of amplitude, perhaps by means of automatic gain control (AGC). However, the phase fluctuation due to the multi-path effect can only be compensated with the help of Eq. 4. Furthermore, the calculated phase differential must be integrated to obtain the phase.

IV. Simulation Results

Using a QPSK signal generated according to the signal model represented by Eq. 13, the performance of the phase estimation method given by Eq. 4 was studied. The real and the imaginary part of the complex wave $s(t)$ and its amplitude are shown in Fig. 2. In Fig. 3, The phase fluctuation without the phase modulation is shown in the top panel. The second panel shows the phase part of the modulated signal $s(t)$. The third panel from the top shows the phase after removing the phase fluctuation shown in the top panel from the second panel. The fourth panel is the same phase as the third panel, but plotted in angle measured within the range of $-\pi$ to $+\pi$, which corresponds to the angle of the randomly generated 4 possible QPSK code words. The bottom panel is the amplitude. The graphs in Fig. 3 demonstrates that even if the QPSK signal is severely affected by flat fading, the demodulated phase signal properly represents the transmitted code words as long as we keep track of the phase fluctuation, exactly.

Following Eq. 4, the magnitude differential $\frac{dA(t)}{dt}$ was calculated as shown in the top panel of Fig. 4. Dividing this by the magnitude shown in the second panel, the third panel showing the phase differential divided by the magnitude was obtained. Then, the Hilbert transform was applied to the graph in the third panel, by using the algorithm discussed in Section II. The graph in the fourth panel is the phase differential signal necessary to calculate the accumulated total phase change. By integrating the phase differential, the total phase accumulated from the onset of the signal was obtained as shown in the bottom panel of Fig. 4. The known phase fluctuation and that of the calculated are overlaid in this panel.

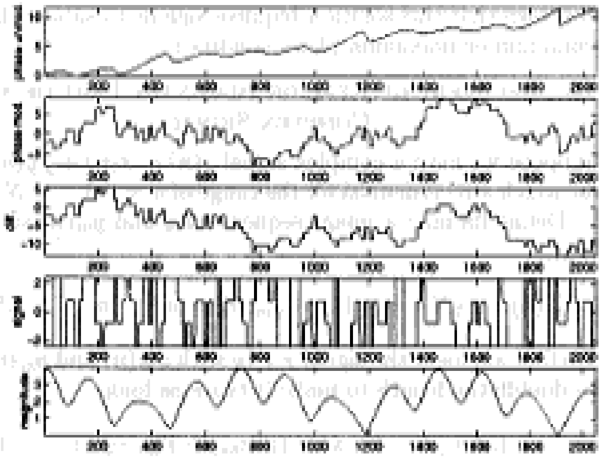

Figure 3: Demodulation process of the QPSK signal under flat fading using the known phase.

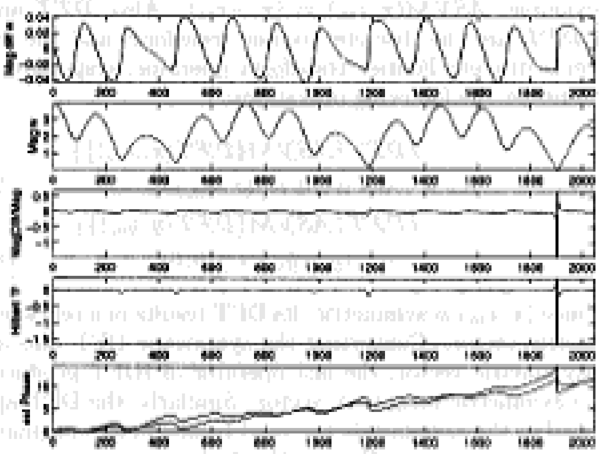

Figure 4: Estimation of the phase under flat fading by means of the Hilbert transform.

The graph that starts out higher, then crosses over to come below the other line is the calculated total phase, in the bottom panel of Fig. 4. The other line which shows a jump around 1900 in the horizontal scale is the known phase. Comparing these two graphs, one can notice a similarity between the two graphs. However, they do not quite agree each other. In this simulation, the minimum phase condition was not checked. The minimum phase condition is, generally speaking, satisfied for a long signal sequence such as this case. So, it is not difficult obtain a reasonable similarity of the calculated phase to the true phase. However, when the signal is segmented in much smaller signal sequences, those sequences that occur at nulls or near zero in the magnitude tends to be non-minimum phase. The reason for this disagreement is due to those hidden non-minimum phase signal segments.

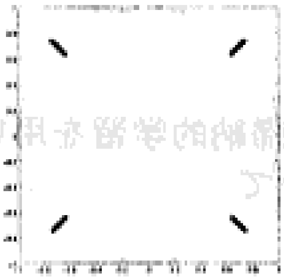

Figure 5: The constellation of QPSK code sequence.

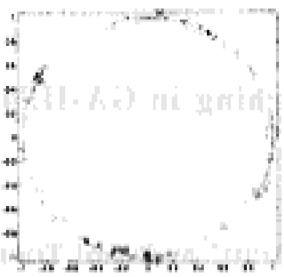

Figure 6: The constellation of demodulated QPSK code sequence after magnitude and phase are compensated.

The constellation of the QPSK code sequence used in the signal is shown in Fig. 5. The constellation obtained after demodulation is shown in Fig. 6, where both of the magnitude and phase are compensated. Because of the discrepancy observed in Fig. 4, the stars in Fig. 6 are distributed to form four clusters.

V. Conclusion

The method to recover carrier phase for constant modulus QAM signals affected by flat fading was discussed in this paper. The basic principle given by Eqns. 3 and 4 is a feasible approach to the problem. Since this problem is of the non-linear channel equalization problem, other methods using higher order statistics such as third or fifth order cummulants are applicable. However, the proposed method is far simpler and requires much less computations. The simulation shown demonstrated that the method works reasonably well even in the severe fading due to the Doppler effect of a moving vehicle. However, the weakness in recovering the imaginary part from the real part by the Hilbert transform is the minimum phase condition. Once the minimum phase condition is violated by a given signal sequence, the result is incorrect. This happens more likely when the magnitude becomes near zero. However, calculated is fortunately the phase differential, which is to be integrated to yield the phase. Ignoring the periods where the magnitude suddenly drops down to near zero merely introduces an phase offset from that time and afterward. It is quite possible to estimate this offset from the rotational shift of clusters in the constellation. The study of adaptive carrier phase recovery based on this notion is currently under way. This paper reported a comprehensive simulation study of the basic principle.

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